2 Fighting Floods with Sandbags

Length: 1 Class Meeting, ˜ 75 minutes

Overview

Summary

This lesson has been prepared as an initial modeling task before mathematical modeling has been defined or introduced. The **general goal** is for students to gain experience using mathematics to analyze a real situation and "discover" mathematical modeling first through this task and formalize the process at a later lesson. The **specific goal** of the lesson is to determine a plausible reason for a discrepancy in two official documents for the number of sandbags needed to build a wall of specified dimensions.

Goals

- Extract information from documents, identify variables, and generate functions to describe the number of sandbags required to build a levee of specified dimensions.
- Apply problem solving strategies.

Materials

- Handout 1: US Army Corps of Engineers Sandbag Article
- Handout 2: Missouri Department of Natural Resources Sandbag Article
- Handout 3: Fighting Floods with Sandbags
- Handout 4: Possible Model

> **Pedagogical Note.** Handout 4 should be printed ahead of time and distributed to PSTs *only after* they go through the modeling process on their own. It may be useful for PSTs to be able to analyze, view, and discuss a model as created by an instructor. Additionally, they will be able to use it as a reference through the remainder of the course.

- Modeling Course Module 1 slides
- Homework Assignment: Sandbags Problem Reflection

CCSSM Standard	Connection to Lesson
MP4: Model with mathematics	This is a real-life situation (not contrived or concocted) that arose from noticing a discrepancy in published documents, leading to the question of what may have caused the discrepancy.
6-EE.9: Use variables to represent two quantities in a real-world problem that change in relationship to one another as dependent and independent variables. Analyze the relationship between the dependent and independent variables using graphs and tables.	As the thickness of the bags and height of the wall change, the number of bags needed to construct a 100 feet sandbag wall will change in relationship to that. PSTs will use variables in an equation to show how these input changes effect the output.
7-G.6: Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	The sandbag walls are roughly triangular prisms built of rectangles. Using these shapes to construct the larger wall allows predictions of size, total sand needed, etc.
A-CED.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*	To find an answer to the sandbags problem, creating a equation and graphing it will help students visualize a solution.
F-BF.1: Write a function that describes a relationship between two quantities.*	The number of sandbags needed is a quadratic function of the desired height of the wall.
G-MG.3: Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost).*	Recognizing that the sandbag wall is a triangular prism, participants can use properties of triangles and rectangles to accurately describe the wall. Additionally, they can use properties of numbers and shapes to properly model the structure.
Concepts Beyond CCSSM	**Connection to the Lesson**
Triangular numbers	The sandbags are arranged in triangles. Using properties of triangular numbers, including the closed formula for finding them, a function can be created to model building a wall of various heights.

Description

Lesson Overview

Briefly re-introduce the Sandbags task (since it was launched at the end of Developing Ways of Thinking for Mathematical Modeling) and pose the problem as described below. Prompt participants to work with a small group. Encourage participants to keep track of their ideas and to think outside the box while creating their model. Participants can use the articles they read for homework as a reference, and should begin with sharing ideas and important information they read.

Pedagogical Note. It is helpful to organize small groups prior to class and facilitate a quick transition into these groups. We recommend changing groups periodically throughout the course so PSTs develop collaboration skills with a variety of their colleagues. Monitor the groups as they work and make selections of groups with various approaches. Urge groups to keep good records of their work and record not only their calculations, but their thinking so they can better reflect on the process.

1. Call on specific groups to share their results and approaches to the sandbags problem.
2. Introduce *Sandbags Problem Reflection*. Instructions: *Write brief answers to the following questions using a different sheet of paper for each item:*
   - *(a) What information did you consider necessary to answer the question in the problem? Where did you find all this information? Did you make any assumptions or choices in the solution?*
   - *(b) What mathematics did you use to solve the problem? How did you select those mathematical concepts?*
   - *(c) Do you feel that the process of solving this problem was linear (step-by-step) or did you find yourself backing up, making changes to your previous decisions and then continuing with the solution? If so, give an example of this.*
   - *(d) Did your solution make sense to you? Discuss the reasonableness of your solution and what you did to improve it if it was not reasonable.*
3. Distribute, consider, and discuss Handout 3 with the mathematical model provided by the instructor. Note that the estimate for the number of sandbags needed depends sensitively on the model parameter for the thickness of the bags. How does this approach differ from the group presentations?
4. As appropriate during the lesson, include some remarks about triangular numbers, and mathematician Johann Carl Friedrich Gauss and his work with triangular numbers. Check that PSTs are comfortable explaining the derivation of a formula for the sum of n consecutive integers.

Introduce the Task

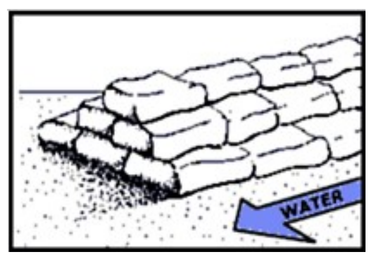
Burlap sacks filled with sand are used to prevent or reduce floodwater damage. Properly filled and placed sandbags can act as a barrier to divert moving water. Sandbags are also used successfully to prevent overtopping of streams with levees.
According to the U.S. Army Corps of Engineers, filling sandbags is a two-person operation. Bags should be filled between one-third to one-half of their capacity. This allows the bags to be stacked with a good seal. The bags should be placed as a pyramid, staggering the position for multiple layers.

Read the articles in Handout 1 and Handout 2

Pose the Problem

Present the task as described in Handout 3: Fighting Floods with Sandbags.

Task. It is important to first get an accurate idea of how sandbag levees are built. Based on the information in the two articles, draw a picture of the wall as you understand it showing the 100 ft length, the height and the width as well as how the bags are arranged within the wall.



Based on the shape of sandbag walls, develop a procedure to estimate the number of sandbags needed to build sandbag walls that are 100 feet long and have heights 1, 2, 3, 4 and 5 feet. The table below also shows estimates from the **Army Corps of Engineers** and the **Missouri Department of Natural Resources** for the number of bags needed. Why do you think their estimates are so different? How do your estimates compare with theirs?

Height of Sandbag Wall	Your Estimate	Army Corps of Engineers Estimate	Missouri Dept. of Natural Resources Estimate
1 foot		600 bags	500 bags
2 feet		2,100 bags	1,000 bags
3 feet		4,500 bags	2,100 bags
4 feet		7,800 bags	3,600 bags
5 feet		-	5,500 bags

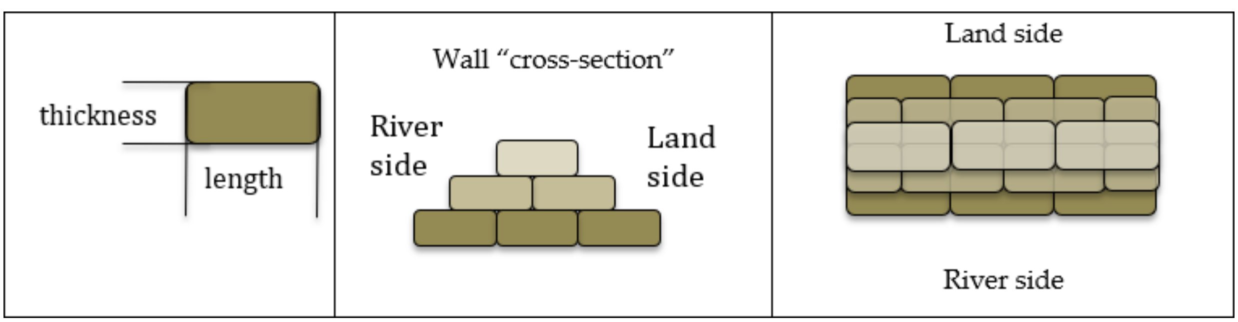
Pedagogical Note. Students can work in small groups, and the instructor can walk around listening to the small group discussions and answering questions. Anticipate questions regarding how to get started, making assumptions and other choices. Encourage students to be creative, as this is likely to be an unusual problem for them.

Sample Approach & Possible Model

Looking ahead at the lesson on the typical elements of the modeling process, sometimes it is difficult to get started on an open-ended problem like this one, especially without prior experience. It helps to start by asking questions like:

1. What information do we need to estimate the number of sandbags needed per 100 feet?
2. Where do we find this information?
3. What do we do with information we need but we can't find?
4. How do we develop a procedure to estimate the number of sandbags?
5. What mathematics might be useful?
6. How do we present this procedure?
7. How do we know how accurate our estimate is?

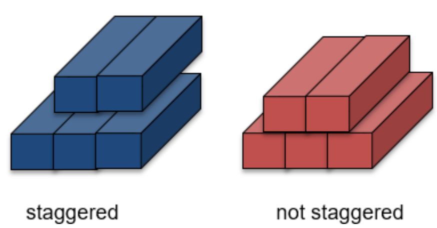
The figure below shows a typical bag viewed from the side (left), an example of the pyramid stacking using 3 layers seen from downstream (center) and a top view (right).



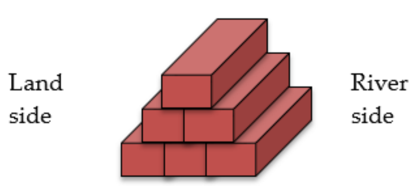
The answers likely to come up are that we need to know the dimensions of each filled sandbag, especially the length (to make sure we cover the 100 feet) and the thickness (to make sure we cover the height of the wall). Also needed is the proper way to stack the sandbags. Most of this information is in the handouts.

After reading the handouts and possibly looking up other sources, we will need to make decisions in order to proceed. Decisions and choices that are stated here as assumptions. Assumptions for the model:

- Each sandbag is 12 inches long when filled.
- The thickness of each sandbag is no more than 6 inches depending on how much the bag is filled.
- The staggering of the bags in the downstream direction does not affect the total number of bags need (see figure below). This is because the staggering results in a half bag sticking out at one end of the wall and a half bag short at the other end, so they compensate for one another.



- The bags are stacked in a triangular pyramid form with 1 bag at the top, 2 bags in the next layer down, and so on (like the figure above). If the pyramid has 5 layers, then the bottom layer will have 5 bags. The figure below shows an example of a pyramid with 3 layers each.



A General Mathematical Model:

Pedagogical Note. PSTs can start with specific numbers and generalize as a later step.

The number of bags needed for a 100-foot long wall is equal to the number of layers in the pyramid times 100 (since the bags are 1 foot long).
Let's define:

- T to be the thickness (in inches) of the bags,
- N_L to be the number of layers in each pyramid, and
- N_B to be the number of bags required to build a pyramid with N_L layers.

Then the total number of sandbags needed for a 100-foot long wall is $100N_B$.
To compute N_B, the number of bags required to build a pyramid with N_L layers, we sum that number of bags in each layer:

$$N_B = 1 + 2 + 3 + \ldots + N_L = \tfrac{1}{2}(N_L)(N_L + 1)$$ [connection to triangular numbers]

Mathematical Note. Triangular numbers come up in this problem. They are obtained from adding consecutive integers starting from 1. For example, 6 is a triangular number because 1 + 2 + 3 = 6. There are formulas to compute these numbers without adding. Notice the pattern

$1 + 2 = 1 \times 3$ $\qquad$ $1 + 2 + 3 = 2 \times 3$

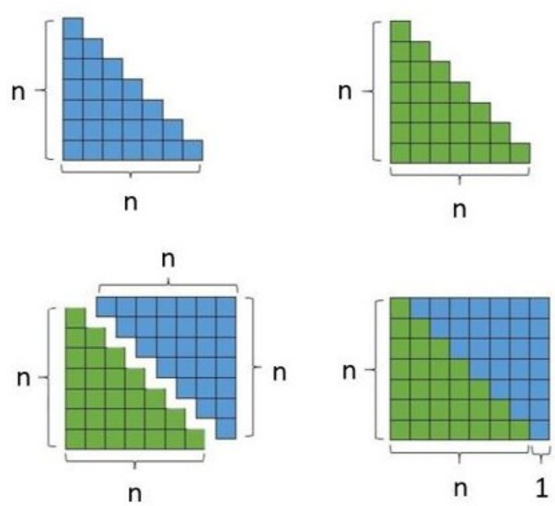
$1 + 2 + 3 + 4 = 2 \times 5$ $\qquad$ $1 + 2 + 3 + 4 + 5 = 3 \times 5$

$1 + 2 + 3 + 4 + 5 + 6 = 3 \times 7$ $\qquad$ $1 + 2 + 3 + 4 + 5 + 6 + 7 = 4 \times 7$

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 4 \times 9$ $\qquad$ $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 5 \times 9$

$1 + 2 + 3 + \ldots + N = \tfrac{1}{2}N(N + 1)$ $\qquad$ $1 + 2 + 3 + \ldots + N = \tfrac{1}{2}(N + 1)N$

There are different ways to prove this formula. Below is a graphical proof.



Therefore, the total number of bags for the wall as a function of the number of layers is

$$\text{total number of bags} = 100N_B = 50(N_L)(N_L + 1)$$

We need to know how many layers are needed. This depends on the thickness of the bags (how much sand is placed in the bags) and the height of the wall we want to build. If the bag thickness is $T = 6$ inches, then we need $N_L = 2$ layers for every foot of wall height. If the bag thickness is $T = 4$ inches, then we need $N_L = 3$ layers for every foot of wall height. In general, if the bag thickness is T inches, then we need $N_L = 12/T$ layers for every foot of wall height.
So, the complete mathematical model for a wall of height H feet is: given a sandbag thickness T (in inches), compute the number of layers with $N_L = (12/T)H$. Then the total number of bags needed is $50(N_L)(N_L + 1)$. The table below is a summary for different bag thicknesses.

Wall Height, N feet	Number of Layers Needed, $N_L = (12/T)H$	Total Number of Bags, $50(N_L)(N_L + 1)$	Wall Height,	Number of Layers Needed, $N_L = (12/T)H$	Total Number of Bags, $50(N_L)(N_L + 1)$
	6-inch thick bags	($T = 6$)		4-inch thick bags	($T = 4$)
1 foot	2	300	1 foot	3	600
2 feet	4	1,000	2 feet	6	2,100
3 feet	6	2,100	3 feet	9	4,500
4 feet	8	3,600	4 feet	12	7,800
5 feet	10	5,500	5 feet	15	12,000
. . .			. . .		
H feet	$2H$	$100(H)(2H + 1)$	H feet	$3H$	$150(H)(3H + 1)$

Wall Height, N feet	Number of Layers Needed, $N_L = (12/T)H$	Total Number of Bags, $50(N_L)(N_L + 1)$	Wall Height,	Number of Layers Needed, $N_L = (12/T)H$	Total Number of Bags, $50(N_L)(N_L + 1)$
	3-inch thick bags	($T = 3$)		2-inch thick bags	($T = 2$)
1 foot	4	1,000	1 foot	6	2,100
2 feet	8	3,600	2 feet	12	7,800
3 feet	12	7,800	3 feet	18	17,100
4 feet	16	13,600	4 feet	24	30,000
5 feet	20	21,000	5 feet	30	46,500
. . .			. . .		
H feet	$4H$	$200(H)(4H + 1)$	H feet	$6H$	$300(H)(6H + 1)$

Conclusion: The number of sand bags needed for the wall depends sensitively on the thickness of the bags once filled with sand. If we assume they are 4 inches thick (green table column) then the numbers match those of the Army Corps of Engineers. If we assume the bags are 6 inches thick (orange table column), then we would need fewer bags and the numbers match those of the Missouri document. The only exception is the 1-foot tall wall. It may be that their estimate uses different assumptions.

Is our model acceptable?

Consider evaluating your model critically. It helps to ask questions like:

- Are the results from my model reasonable?
- Are there other variables that should be taken into account?
- Are the assumptions made acceptable or should they be changed?
- What if the bag thickness in 5-inches? Then $N_L = 12/5 = 2.4$ layers!

Since both estimates, from the Army Corps of Engineers and from the Missouri document, are obtained from our model by just changing one parameter (the thickness of the filled bags), we can conclude that we answered the question satisfactorily. The fourth bullet is something we didn't think about before, but it can be accounted for by simply redefining N_L to be "the smallest integer greater or equal to $12/T$." Symbolically, this is written $N_L = \left\lceil \frac{12}{T} \right\rceil$. For instance, $\left\lceil \frac{12}{T} \right\rceil = \lceil 2.4 \rceil = 3$

In-Class Resources

Handout 1: US Army Corps of Engineers Sandbag Article

Sandbagging techniques
Brochure from *US Army Corps of Engineers, Northwestern Division, 2004*

Citation:
U.S. Army Corps of Engineers, Northwestern Division. (2004). *Sandbagging techniques* [Brochure]. Retrieved from `usace.contentdm.oclc.org`

Handout 2: Missouri Department of Natural Resources Sandbag Article

How to construct a sandbag emergency levee
Article from Missouri Department of Natural Resources *by Carol S. Comer, March 2014*

Citation:
Comer, C. S. (2014, March). How to construct a sandbag emergency levee. *Missouri Department of Natural Resources*, retrieved from `dnr.mo.gov/pubs/pub2217.htm`.

SANDBAGS PROBLEM REFLECTION

Write brief answers to the following questions using a different sheet of paper for each item:

1. What information did you consider necessary to answer the question in the problem? Where did you find all this information? Did you make any assumptions or choices in the solution?
2. What mathematics did you use to solve the problem? How did you select those mathematical concepts?
3. Do you feel that the process of solving this problem was linear (step-by-step) or did you find yourself backing up, making changes to your previous decisions and then continuing with the solution? If so, give an example of this.
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